

Extended summary

Structural glass between design, tests and models

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Abstract. The research activity has been configured as an investigation on structural glass. Glass is a new material if placed in the field of structural materials, because until recently it was used mainly for glazing and/or curtain walls. Instead, in recent years, we have seen that the glass is increasingly used for structural parts, such as flooring, staircases, balustrades, canopies, roofing, etc. Our investigation, however, focused on the characterization of structural glass as widely as possible, looking from the point of view of the design, from the one of the material testing and from the one of the mathematical models.

Keywords. Asymptotic expansion, dynamic analysis, structural glass.

1 Problem statement and objectives

1.1 Introduction

Over 250 years ago, glass was already recognized as a unique material, but it was impossible to image its use in modern building of our day. Architecture Week stated in 2002 that, "Laminated safety glass frees architects from strict reliance on opaque structural materials" [1]. Glass also frees designers from the confinements that separate people from the environment. In ways never seen before, it can simultaneously protect us from the environment and enhance our relationship with it. How many other transparent materials have the load-bearing capacity of glass? To accept glass as a structural material, we have first to free ourselves from the settled notion that glass is a delicate ornament.

Architects love glass because it does not obstruct a view or visually interrupt a room. Structural engineers should love it because when theoretically compared to steel, it can carry two times the tension load. Moreover, because glass is the most recycled material in the world and the raw material is widespread and non-detrimental to the environment. However, theory and practice are two different things. While glass would win a theoretical competition in order to find the best building material, it would fail in a practical contest. In order to gain widespread acceptance as a structural material, glass has to overcome its social and physical limitations.

The social limitations of glass include the psychological effects of having no privacy and the stigmata that glass is fragile and weak. As for the perception that glass is fragile and weak, this can be overcome with education. Actually, glass is very strong and versatile, but most of the people feels glass as dangerous and the building with it very risky. Because of this risk many owners do not ask for it and many contractors do not agree to build employing it. Actual physical limitations do not hinder the growth of the use of glass as a structural material. The main weakness of glass is its brittle nature. Glass is a brittle material because it fails in tension and it does not in shear, and it deforms very little before it breaks. Finally, glass develops forked fractures due to internal stresses. When a small fracture develops in one part of the glass, immediately it begins to grow up and it leads to the total rupture of glass, splitting small pieces all around [2].

Glass behaves in a crucially different way from other, more familiar, structural materials, such as steel and, due to this fact, structural engineers usually design structures concentrating their attention to limit stress beneath the maximum yield stress. In this way, stress concentrations and lack of ductility do not represent a problem. Instead, structural engineers cannot ignore lack of ductility and stress concentrations when they design with glass [3]. Another difference with usual structural materials is the possibility that glass panes can deflect more than its own thickness and this fact implies that designers have to use large deflection theory, which is an unfamiliar field. One consequence is that design stresses obtained by the use of the small deflection theory are generally larger than realistic design stresses obtained by the use of the large deflection theory.

In our work, we try to explore glass behavior, using different theories, models, tests, designs, in order to exploit its own characteristics.



1.2 Glass as structural material: design concepts and structural elements

The behavior of brittle glass, with its strong randomness of strength characteristics, has carried on the introduction of the fundamental concepts of design which are hierarchy and redundancy. The first concept assigns importance indices to different structural elements, while the second concept ensures an appropriate safety level of the entire structure in case of failure of the single glass element.

This approach is already used in aeronautics, where is accepted that one element can fail without its fail causes the breakage of the entire structure. This approach is called "fail-safe" and it has to be used in glass design instead of the classical "safe-life" [4] used in usual design with concrete or masonry for example. The application of hierarchy and redundancy compensates for the lack of material ductility at the level of the entire structure.

As seen above, the concept of hierarchy assigns a certain importance level to each structural element. From this point of view, glass structural elements are however secondary elements, but they can be subdivided in first class and second class elements. The first class comprehends elements like safety barriers, balustrades, floors and fins, while the second one comprehends other elements like infill panels and generally non-structural elements.

The concept of redundancy can be subdivided in three different typologies: structural, section and system. The first one represents structural capacity to redistribute the total stress state in all its parts, so that failure of one single element not implies the rupture of the entire structure. The second one is the element section capacity to maintain residual strength capability in case of rupture of one of its parts. It is important to notice that a low material stress level do not provide redundancy to the section, on the contrary to what happens to steel or concrete sections. This is because of the bad tenacity of glass: in fact, once the fracture is triggered, failure of monolithic glass is immediate and catastrophic. The third typology, i.e. system redundancy, means that the structure has the capacity to redistribute loads using alternative mechanisms from the design ones, after failure of single part.

Some important glass structural elements are: beams, fins, columns and walls, stairs and floors and point-supported glass. There are also some episodic projects in which glass is pushed in direction different from usual, like glass masonry in [5] or in the "Glass House" in The Netherlands. In all these realizations, we need great care in the design and manufacture of glass and of the relative joints. The material which is in direct contact with glass has not to be soft not to cause concentrations of stress on glass and it has to be elastic enough in order to accommodate possible different thermal behavior between glass and the restrain element.

Glass beams are exactly what the name implies: beams made by glass. They are loaded by bending horizontal elements, generally simply supported or cantilevering and limited to the length of a single piece of glass that can be manufactured. A notable exception to this is the entrance canopy into the Yuraku-cho underground station in Tokyo, designed by Drewhurst McFarlane in which the cantilevering glass beams are composed by four panes of glass bolted together. Like glass beams, glass fins are exactly what the name implies: fins made by glass. They are loaded by bending vertical or sloping elements used to support façades and to help to resist against wind or other lateral loads. The fins are composed by glass using structural adhesives or bolting. Generally they are bonded in the same way as beams.

Glass columns and load-bearing walls are rather rare. The reason is that engineers usually prefer to design compressed elements composed by a material that works in the best way in this field. Now the question we pose is the following: if glass works better when com-



pressed, why engineers do not use it? Again, the problem is the brittleness. In fact a glass column is an element that can fail suddenly, without warning. This means that a structure has to be able to cope with the loss of a column avoiding an entire collapse, following structural hierarchy and redundancy as seen above.

Glass stairs, floors and bridges use glass as a walking surface. Use of glass in these structural elements allows to create an open space view because the physical limit of floor, on which people walk, is transparent. It is useful in case of an ancient or delicate floor on which people cannot walk not to damage it; we can cover it with glass which permits the same to see the ancient floor. Because it represents a walking surface, glass flooring needs particular attentions when designed. Usually we have the use of minimum three panes of glass, because if one fails the other two panes remain and they sustain loads. A famous glass bridge is the "Grand Canyon Skywalk", on which people can feel the emotion to walk over the most important canyon of the world.

2 Research planning and activities

As we can seen, glass is a complex material and its behavior has complex features, too. Its use in structural applications has to be studied deeply, because its brittleness can influence the response of the entire structure to external loads and make useless a good design job. Standards present in European regulatory system give very important tools to cope with design problems of glass and they have to be chosen in every design activity. In our research work we addressed not only the design approach. In order to look at glass as a structural material, we can use multiple strategies, which are useful to understand every own feature of the material and of its behavior.

This approach is valid for every structural material, because a deep knowledge of a material allows a better structural use of it. For classical materials this approach is already consolidated in literature and structural designers have many data and instruments to cope with design problems. In case of particular materials, such as structural glass, literature and data are fewer both from a quality and from a quantity point of view. Nevertheless in contemporary design, structural glass is employed extensively for its aesthetic appeal. Its use, beyond its intrinsic drawback constituted by brittleness, is possible if and only if its mechanical behavior is understood, completely or at least up to a certain extent. Fixed these points, various approaches have been started with the aim of improving the knowledge of structural characteristics of structural glass. Here these approaches are presented at a same importance level, independently from the order of publication, because the three strategies contribute equally to the objective.

From the classical point of view of the design, we have chosen the typology of the glass truss from the many possible typologies. The reason is that it is a not very common structural element in architectural design, but it has some good characteristics that could be used more often in projects. The basic idea is to design a glass truss that can be transported easily and adapted to different configurations. Using glass and stainless steel, the truss exploits the mechanical characteristics of the component materials: steel carries tensile stresses and glass carries compressive stresses. The glass truss is designed using the necessary Standards and it is verified both in static and dynamic conditions, because of the possible uses of it, i.e. as main structure of a roof or a continuous façade. This design activity is merged in an Italian patent and here some technical details are introduced.



The second typology of investigation used is focused to comprehend the mechanical behavior: it regards structural measurements. The aim is to obtain a valid comparison between experimental results and simple analytical or Finite Element models. In particular, dynamic problems are addressed. The objective of the dynamical tests is to determine the characteristic modal parameters of the glass elements, such as natural frequencies, modal shapes and modal damping to characterize the response of the system. In this way we can focus on some problems like various thickness of laminated glass and important features of interlayers. Also static measurements are performed, in particular we planned compression tests to determine a kind of "ductility" in beams of structural glass and we have obtained some initial interesting results.

Last but not least, we present a theoretical approach. By using the technique of asymptotic expansion, it is investigated the influence of interlayer in dynamic conditions. The analysis searches the natural frequencies and the field of displacements of a laminated glass beam and it focuses on the role of the mechanical characteristics of interlayer. The analysis of two-layers laminated structure is then carried on. Different asymptotic developments of frequencies are used, while displacements are not scaled. From these three different developments of frequency, it is found that each one represents one different wave which runs inside beam and determines different natural frequencies.

3 Analysis and discussion of main results

3.1 Design issue

The truss we designed, shown as example in Figure 1, is interesting from both a structural and a mechanical point of view, but it is also interesting from an economic and innovative point of view, as we explain below. For this reason, an Italian Patent was born from this design work: it was registered on the 8th September 2009 at n. BO2009A000571, with Faraone S.r.l. as applicant and Stefano Lenci and Laura Consolini as inventors.

The invention was born to try to exploit peculiar characteristics of both glass and stainless steel and to overcome some drawback of the classical trusses. One drawback is that they are very bulky during storage and transport; another drawback is that they are sized in function of span and loads to cope, generally designed from time to time, depending on the specifications; another drawback is that they interfere with lighting, especially with the natural one in case of sustaining roofs and façades. This invention, unlike the examples of glass trusses shown above, can be adapt to many different situations. In fact, the truss has three specific features that characterize it and adapt it in different cases:

- 1. assembling the base module, we can obtain multiple lengths and different configurations of the truss;
- 2. the final geometric shape of the truss (linear or curve) can be varied depending on the use and on the loads of each situation;
- 3. the base module of each truss is transportable in a bag of contained dimensions.

These reasons give to the project an important innovative feature. In this list, the third point is the last but not the least: on the contrary, it is the more appealing idea behind this project, because gives to it the features of total adaptability, of assembling easiness and of industrial production.

Summarizing, three are the major purposes of this invention:



- 1. it can be easily folded and assembled with virtually no risk of errors in the installation site, allowing easy and cheap storage and transport;
- 2. the base module can be composed to obtain different configurations;
- 3. it consists of transparent elements, which minimize light absorption.



Figure 1. Linear configuration of the glass truss.

3.2 Tests issue

We perform both static both dynamic tests. Static tests are simple compression tests and we use samples of laminated glass which overall dimensions are: $1000 \ge 80 \ge 33$ mm.

Figure 2 shows the stress-strain paths of a sample and Table 1 shows the maximum loads reached.



Figure 2. Stress-strain path of the laminated glass sample.

Analyzing these graphics we can notice some important characteristics of the breakage behavior of laminated glass. The linear elastic behavior is evident, in fact the first part of the graphic is linear, according to the elastic modulus of glass. The rupture is not brittle, because the sample deforms without any increasing of the load.



	1		
	Load (kN)	Vert. displ. (mm)	Horiz. displ. (mm)
n.0	51.16	5.49	20.17
n.1	55.30	6.03	19.75
n.2	56.52	4.78	9.08
n.3	48.72	6.74	12.54

Table 1. Results of compression tests.

There is a significant deformation in horizontal direction, and this fact attests the buckling mechanism of failure, excluding the breakage due to maximum compression strength. The difference between monolithic glass and laminated glass is immediately visible considering the ending part of the linear elastic branch: in our case, the breakage is not brittle and it does not happen suddenly at the end of the linear branch. Here we can notice the presence of a small landing before breakage. This landing appears after reaching the maximum load which the sample is capable to support, and it is not perfectly horizontal, but it comes down a little before breakage. Thus, we can observe a sort of "plastic" behavior of laminated glass.

Dynamic tests are planned and performed to increase knowledge and confidence with overall behavior of laminated glass. The aim is to obtain a valid comparison between experimental results and simple analytical or Finite Element models and to try to comprehend the interlayer behavior.

Laminated glass in dynamic conditions is studied in its mechanical aspect, in order to determine its modal parameters and to construct models useful in future (in the same direction, there are already some works like [6, 7] where mixed numeric-experimental methods are use to investigate damping or viscous properties of composite laminates beams or plates). Models are useful to understand the long-term behavior of structural laminated glass: how mechanical properties of glass and of the interlayer could modify themselves under the influence of time and temperature, that are very important environmental factors always present in architectural and structural design applications. Having available interpretative models and experimental data we can use non-destructive techniques to monitor laminated glass elements which could be also large, such as curtain walls, roofing or flooring.

The first set of measurements is performed in the Department of Architecture, Buildings and Structures of Polytechnic University of Marche and it is planned to extract the modal parameters from the samples analyzed, i.e. natural frequencies, ω , modal damping, ξ , and modal shapes, ψ . The purpose of these measurements is to extract modal parameters in the most simple way, obviously to comprehend the behavior of laminated glass, but also to test the measurement instruments in terms of reliability and accuracy.

The second set of measurements performed in the Department of Mechanics of Polytechnic University of Marche is composed by a PolytecTM Vibrometer and the relative integrated signal processing system. Difference with the first set is mainly in the number of modes extracted and in the range of frequency investigated.

Results of the first set are fully included in those ones of the second set, so we discuss here in complete way only results of the second set, as said above. The agreement between measurements found by the first set and the second set is very good and this fact allows us to choose the set we prefer.

Before and after performing measurements, we planned to do different models in order to obtain a self-validation of both measurements and models. Then we compare them and



we try to find some possible explanations in case of discrepancies. We use both analytic and numeric models to perform some comparisons between them and measurements.

Table 2 shows what we obtain for one of the tested sample. We collect a table like that for each sample and we discuss results.

Mode	Exp. (Hz)	Num. (Hz)	Ratios	Anal. (Hz)	Ratios
1	117	108	1.08	111	1.05
2	307	279	1.10	306	1.00
3	576	502	1.15	600	0.95
4	905	743	1.22	992	0.91
5	1282	1052	1.22	1482	0.86
6	1702	1417	1.20	2071	0.82
7	2159	1838	1.17	2757	0.78
8	2668	2317	1.15	3541	0.73
9	3216	2854	1.13	4423	0.73
10	3749	3447	1.09	5403	0.69

Table 2. Summary table of experimental and model frequencies.

For the single-layer sample, our tests on this sample confirm what obtained by theory and we can say that single-layer sample has a clear beam behavior, as expected.

Let us consider now Table 2 with results of the double-layer sample. We have performed both analytic and numeric models, so in the center there are two columns with numeric frequencies and the relative ratios between them and the experimental ones. On the right there are other two columns with analytic frequencies and their relative ratios. As we can see, all ratios are comparable and they give a good validation of measurements. One aspect to notice is the difference of ratios between numeric and analytic models. In the first case, we have difference less or equal than 10% only at the first two modes, then the ratios increase above 15-20%. In the second case, the difference is 5% at the first and the third modes, 9% at the fourth mode and at the second mode we have no difference, then the ratios increase above 14%. This fact can be explained considering the more geometric and material complexities introduced by the add of one glass ply and of the interlayer in numeric models. We can also notice that ratios with numeric frequencies are systematically higher than unit (it implies that experimental frequencies are higher than the numeric ones) while ratios with analytic frequencies are systematically lower than unit at modes greater than second (it implies vice versa that experimental frequencies are lower than analytic ones). This is due to presence or not of interlayer in models: analytic one considers glass beam as a monolithic beam of double thickness while numeric one introduces a thin layer of material between the two glass plies. As said in the previous section, G used in FE model is 20 MPa and, summarizing, the double-layer sample behaves similarly to a sample composed only by glass that has double thickness, in particular at the first modes. Both models show the same behavior, the little differences are due to numeric model features.

Eventually, we consider the results of the triple-layer sample. Like the previous results, here we compare the experimental frequencies with numeric and the analytic ones and the relative ratios. On the contrary from the previous situations, here no ratios are comparable: analytic frequencies are significantly far from measurement ones. Thus the triple-layer sample can not be assimilated to a monolithic one with triple thickness. On the other hand,



numeric frequencies are well suited with the experimental ones, as we can see looking at ratios between them: in fact the difference is less than 4% at the first three modes, less than 10% till the ninth mode and then it reaches 12%. The problem lies in the fact that the frequencies derived by numeric model are obtained using the ordinary value of G, i.e. 0.6 MPa, and not using the same value implemented in the double-layer model (20 MPa). Thus we find a difference of behavior between the double-layer sample and the triple-layer sample: we expected that the frequency, qualitatively, increases, but this fact does not happen. We perform some consideration in explaining this fact and the most reliable regards the constitutive law of interlayer materials. Although with difference, all the typologies of interlayers show an hardening behavior. In the double-layer sample amplitude of deformation is higher than in the triple-layer one because interlayer lays exactly on medium plane of vibration, i.e. the more stressed. In this way, the shear modulus involved is higher than the one involved in smaller deformations like the those ones present in triple-layer sample. Frequency is directly connected to G, so with increasing it frequency increases. This explanation is however not complete, because in our vibration test deformations are always small and not so highly different to justify so otherwise values of G.

3.3 Model issue

We have performed different asymptotic analyses (see for example [8]) to obtain models on dynamics of laminated glass beams without use of a priori assumptions, that are sometimes mechanically unjustified in classical formulations but are used to reduce the number of unknowns and simplify the three-dimensional elastic problem.

We obtain the limit model for p = 2 corresponds to flexional vibrations propagating along the x₂-coordinate, i.e. along the length of the beam and the simplified model homogenizes the three-layer beam into one consisting of just one layer. Thus the influence of the middle soft layer is negligible in low frequencies vibration. The natural frequency at the first order is expressed by Equation (1) below:

$$\omega_{k,2} \simeq (2k+1)^2 \frac{\pi^2 h}{4L^2} \sqrt{\frac{3\lambda^2 + 9\mu^2 + 13\lambda\mu}{3\rho(\lambda + 2\mu)}}$$
(1)

Then for p = 0 corresponds to axial or stretching vibrations propagating along the x1coordinate, i.e. along the length of the beam and the simplified model reduces the threelayer beam to one consisting of just one layer. Thus in the limit model we cannot perceive the presence of the soft thin adhesive layer and its effects on the dynamics of the beam. The natural frequency at the first order is expressed by Equation (2) below:

$$\omega_{k,0} = \frac{k\pi}{L} \sqrt{\frac{4\mu(\lambda+\mu)}{\rho(\lambda+2\mu)}} \tag{2}$$

These results are important because we have a starting point for future developments. In particular, from the mechanical point of view we will study laminated objects with more plies or make some comparisons between different interlayers, and we will perform also experimental tests to compare theoretical results with experimental ones. From the theoretical point of view indeed, we will evaluate the convergence of the solution of the physical problem towards the solution of the limit problem.



4 Conclusions

We conducted an investigation into the structural glass as wide as possible, touching on various themes and trying to raise many issues to make the glass more and more similar to a building material for all purposes.

Glass is an innovative material if compared to the other more familiar structural materials, because until recently it was used mainly for glazing or for curtain walls. Instead, in recent years, we have seen that the glass is used for structural parts, such as floors, staircases, balustrades, canopies, roofs, etc. In all these typologies, the glass has to behave as a building material for all purposes, such as concrete or steel. Looking at glass from the point of view of the structural material, it is evident the need and the utility of regulations in the calculation of structural glass. So we have to consider the standards present in European and Italian systems. The European system is more detailed than the Italian one, in which the most of indications are due to the UNI System. For this reason, in recent years, the need in Italy for comprehensive legislation on the structural glass (as already present in many European countries) is very urgent. Thus, to elaborate a standard unified document, a voluntary committee has set up at the CNR for the drafting of these regulations, and here we shared the "models" group.

In our work on structural glass, we faced with different issues concerning with it: we look at it from the point of view of design, then we use the point of view of experimental tests and eventually we treat it using a theoretical point of view.

The design has paid attention on the development of a structural element, easy to produce and sell. We choose to design a glass and stainless steel truss. Key features of this element are: modularity, the possibility of curve configurations and the portability. The first concept concerns with the fact that the beam consists of a base module repeatable until a total length of 6.90 m; the second one regards the fact that the elements of the basic module can rotate mutually; the third one concerns the fact that the module "flattens out" turning the elements and it can be transported more easily. The beam has been studied in various configurations and both in static and dynamic conditions. At the end of the design process it was merged in an Italian patent. In future, we hope it will be possible to produce a prototype of the truss to achieve results comparable with experimental models and to check its overall behavior.

Regarding the second issue addressed, we conducted experimental tests both in static and in dynamic field. In statics, we have performed simple compression tests, first without instrumentation for displacement data and then adding them. In this way we could analyze the glass failure mechanism: we can notice that our samples of laminated glass (consisting of three layers of glass) do not undergo brittle failure, but in the stress-strain graph a sort of plastic landing appeared, due to presence of interlayer. Since now, the conducted treatment of data concerns only the maximum load reached, while in the future we have already suggested the use of Koiter's model or Frostig's model to retrace the path of buckling.

In the dynamic tests performed we have used accelerometers and manual hammering in the first case and we have used a laser vibrometer in the second case. The main aim of these tests was to understand the behavior of the glass beams and of the interlayer in dynamic conditions. We obtained the modal parameters, such as the natural frequencies, the modal damping and the mode shapes using different methods of dynamic identification. Three different typologies of samples have been tested: a monolithic glass, a laminated glass composed by two layers of glass and a laminated glass composed by three layers of glass. The monolithic glass behaves as a beam in free vibration, as expected. The two-layer



sample behaves at first modes as if the interlayer achieves a perfectly rigid connection between the layers of glass, thus making the behavior similar to that of a monolithic beam. The three-layer sample has some anomalies of behavior, because its frequencies are lower than those of the two-layer sample, instead of higher. We searched in literature some possible explanations for this phenomenon, understanding that the "temperature" factor is one that most affects the interlayer behavior. The three-layer sample was the only one that undergo cycles of considerable temperature variations and it is possible a behavior change due to temperature. In future, we hope to conduct other tests from SHM point of view, "heating" and "cooling" the sample and recording the changes of the shear modulus of interlayer. We can also think to conduct pure shear tests to reconstruct stress-strain diagram of interlayer, checking a possible hardening behavior.

The treatment of laminated glass from theoretical point of view was the last exposed issue. We use the technique of the asymptotic expansion. We obtained the natural frequencies of a multi-layer element composed by a linear elastic materials with strong contrast in mechanical properties, such as glass and the relative interlayer. We described the limit behavior of the multi-layer using a small parameter, ε , identifying the pulsations at low and medium frequencies. This was achieved using two different asymptotic expansions for the pulsation ω . Future developments will include the modeling of a multi-layer composed by more layers or laminated with materials with different mechanical properties. In addition, we could also compare the theoretical models with some dedicated experimental campaigns or, from a purely theoretical point of view, evaluate the convergence of the solution.

Our investigation on structural glass touches some different and interesting issues, but however they are only a few in comparison with the many possible. Carrying on these issues, we can improve our knowledge on behavior of structural glass and we can use it more and more consciously.

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